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Mechanics of Materials & Structure

Structure :

Structure is the arrangement of structural member. Structural member means beam, column, slab etc. Structure resist & transfer load to the support. Example: Building, Bridge, Dam, tower, etc.

Stress (σ)

The internal resisting force per unit area by the body against the deformation is called stress. Denoted by σ. Unit: N/m^2

Mathematically:

$$\text{Stress}(\sigma) = \frac{\text{Internal resisting force}(R)}{\text{Cross-sectional area}(A)}$$

→ Numerically: Internal resisting force = Applied force

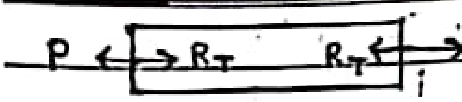
→ It is originates only in deformable body.

→ $1\,N/m^2 = 1\,Pa$
→ $1\,kPa = 1\times10^3\,Pa$
→ $1\,MPa = 1\times10^6\,Pa$
→ $1\,GPa = 1\times10^9\,Pa$

Types of stress

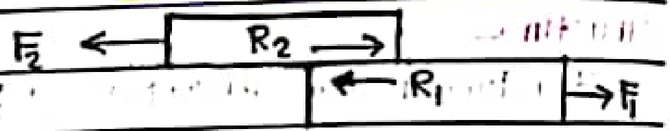
1. Direct stress
   - → Normal stress or Axial stress
     - → Tensile stress
     - → Compressive stress
   - → Shear stress
   - → Bearing stress
2. Indirect stress
   - → Bending stress
   - → Torsional stress
3. Combined stress

Normal stress → The stress acting perpendicular to the area of cross-section of the member.
→ It may be either tensile or compressive.



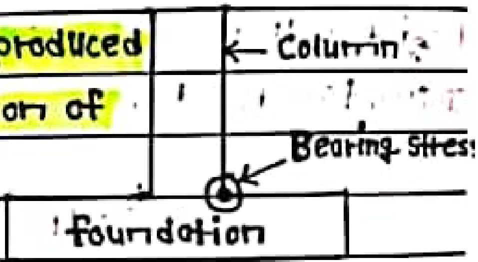
fig; Tensile stress   fig; Compressive stress

Shear stress (τ) → The stress acting along or parallel to the cross-section of the member.



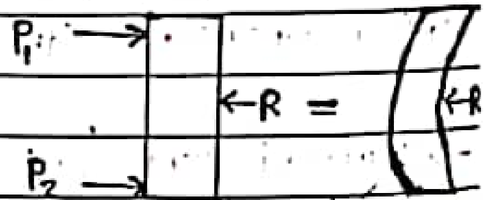
Bearing stress → The stress produced at the state of load transfer portion of structure.

→ Example; Column to foundation load transfer.



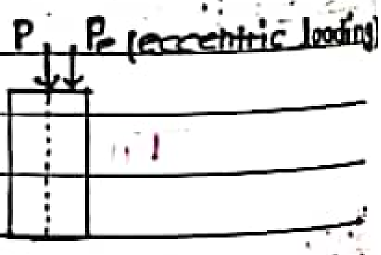
Bending stress

→ The stress produced due to bending moment.



Torsional stress

→ The stress due to eccentric loading.



Strain (ε)

The ratio of change in dimension to original dimension. Denoted by ε, unitless.

Mathematically; $$\text{Strain}(\varepsilon) = \frac{\text{change in dimension}}{\text{original dimension}}$$

Types of Strain

① Longitudinal / Linear / Normal / Axial Strain

→ It is the longitudinal direction strain due to the longitudinal directional load.

→ It may be tensile or compressive strain.

→ $$\text{Longitudinal strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } (l)}$$

② Lateral / Transverse Strain

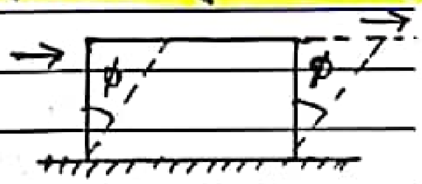
→ It is the lateral direction strain due to the longitudinal directional load.

→ $$\text{Lateral strain} = \frac{\text{Change in diam } (\Delta d)}{\text{Original diam } (d)}$$

③ Shear Strain

→ It is the lateral directional strain due to the lateral directional load. → It is the strain that change the angle of an objects.

→ ϕ = shear strain.



④ Volumetric Strain

$$\varepsilon_v = \frac{\text{Change in Volume } (\Delta V)}{\text{Original Volume } (V)}$$

→ It is the ratio of change in volume to original volume.

Important points

Rigid Body	Deformable Body
→ Not change shape & size with respect to load	→ Change shape & size under load.

Poisson's Ratio

→ It is the ratio of lateral strain to longitudinal strain.

→ It value lies between −1 to 0.5

→ For engineering material it value lies 0 to 0.5.

→ Its Value may be negative too. → $1/m$
→ It is represented by μ(mu). → Unitless quantity
→ Mathematically: $\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

S.N	materials	Value of Poisson's Ratio
1	Steel	0.25 — 0.33
2	Concrete	0.08 — 0.18
3	Clay	0.30 — 0.45
4	Cast iron	0.21 — 0.26
5	Rubber	0.50
6	Wood	0.25

Note: Poisson's ratio for mild steel = 0.25
for Stainless steel = 0.33
for M_{20} grade of concrete = 0.15

Hooke's Law

It states that "stress is directly proportional to the strain upto certain limit. → Robert Hooke → find in 1678

Mathematically: $\sigma \propto \varepsilon$ or $\sigma = E\varepsilon$

Where, E = young modulus of elasticity.

$= \frac{\sigma}{\varepsilon} = \frac{N/m^2}{\text{no unit}} = N/m^2$

Note:

$E_s = 2.1 \times 10^5 \, N/mm^2$ for mild steel
$E_s = 2 \times 10^5 \, N/mm^2$ for only steel
$E_c = 5700\sqrt{f_{ck}}$ for concrete IS456 : 1978
$E_c = 5000\sqrt{f_{ck}}$ for concrete IS456 : 2000
Where; f_{ck} = Characteristics strength of Concrete.

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Stress – Strain curve for Ductile Materials

Ductile materials undergo a large amount of deformation before failing. the following figure shows the stress – strain curve for a ductile specimen.

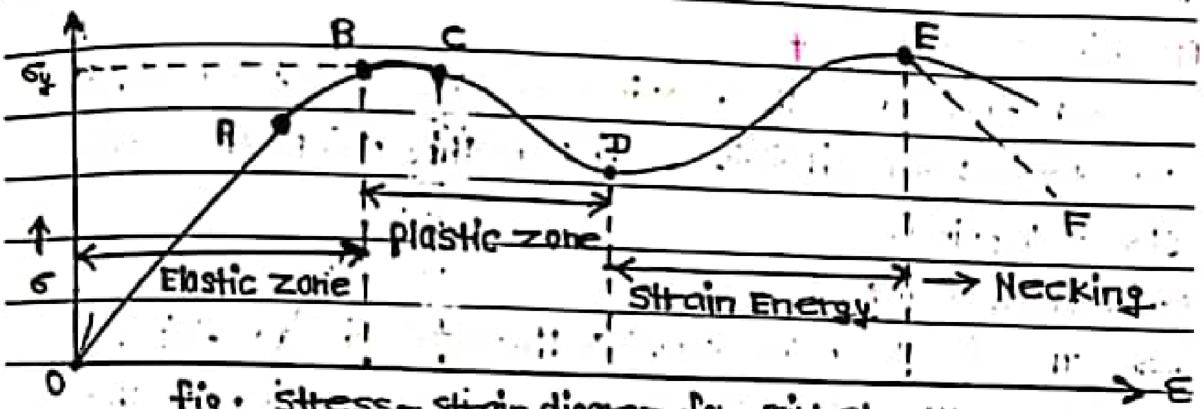

fig: stress – strain diagram for mild steel.

Where: A = proportional limit

B = Elastic limit

C = Upper yield point

D = Lower yield point

E = Ultimate stress point

F = Breaking point

① Proportional limit

→ upto this limit, stress is directly proportional to the strain.

→ specimen obey's hooke's law. → OA is straight line.

→ Beyond this point, the stress not proportional to strain.

② Elastic limit

→ upto this limit, the material is said to be elastic.

→ Specimen regains its original shape & dimensions after the removal of the external load.

→ There is no residual deformation is seen in that specimen, on removal of the load.

→ After this point the material's said to be plastic.

③ Yield Point

· Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

④ Ultimate stress

It represents the maximum stress that a materials can take before it fail. → After this point, the curve starts dropping.

⑤ Breaking Point

→ This is the point at which the specimen fails.

→ After the ultimate stress point, necking of the specimen takes place, which causes a loss in the load carrying capacity of the specimen & ultimately causes it to fail.

Necking

Properties of Materials

① Strength

→ Strength is the ability of materials to withstand various external forces.

→ It is usually described as tensile strength, compressive strength & shear strength etc.

② Ductility

→ property of material by virtue of which material undergoes plastic deformation without failure under tensile force. → Tensile test is done for ductile materials.

→ Carbon content in steel is determine the ductility of steel. Increase in carbon content in steel increase the strength & hardness but decrease ductility & toughness

→ Fe_{250} is more ductile then fe_{500}.

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→ Ductile material fail slowly giving warning sign.

→ Long permanent elongation due to tensile force before fracture.

③ Britlleness

→ It is the opposite of ductility.

→ Brittle materials fails suddenly without warning sign.

④ Elasticity

→ properties of the materials to return back in orginal position after the removal of load.

⑤ plasticity

→ properties of the materials do not return back in orginal position after the removal of load.

⑥ creeps

→ Time dependent Continuous deformation under Constant load.

⑦ Hardness

→ It is the ability of materias to resist abrasion or cutting.

→ Hard materials resist scratches by friction with another body.

⑧ Toughness The resistance of material to fracture by bending, twisting, impact of load is known as toughness.

⑨ Fatigue permanent internal structure damage due to repeated load.

⑩ Malleability permanently extend into sheets without fracture when rolled & hammered.

⑪ Tenancy Ultimate tensile strength of materials.

⑫ Stiffness The load required to produce unit deflection.

⑬ Endurance Limit The stress below which a material has a probability of not failing under reversal of stress.

⑭ Resilience Strain energy stored with the elastic limit.

⑮ Proof Resilience maximum strain energy stored at elastic limit.

⑯ Modulus of Resilience The proof resilience per unit volume.

$\boxed{MOR = \frac{1}{2} \times \sigma \times E}$

Thermal stress

→ Stress produced due to change in temperature.

→ if one end is free, change in tempr does not produces any stress.

→ if both end are fixed, increase on temperature produce compressive stress whereas decrease in temperature produces tensile stress.

$\boxed{\text{Thermal Stress } (\sigma_t) = \alpha \cdot dt \cdot E}$

Thermal strain

→ Change in dimension of a body due to temperature with respect to it's orginial dimension is called thermal strain.

→ Mathematically; $\varepsilon_t = \frac{dl}{l} = \frac{\cancel{l} \cdot \alpha \cdot dt}{\cancel{l}} = \alpha dt$

Where; α = coefficient of thermal expansion.

Note: α for steel $= 11 \times 10^{-6}/^0C$

Concrete $= 12 \times 10^{-6}/^0C$

Copper $= 17.5 \times 10^{-6}/^0C$

Aluminium $= 21 \times 10^{-6}/^0C$

Relationship between three elastic modulus (E, G & K) & Poisson's Ratio (μ)

For normal stress & strain. $\sigma \propto \varepsilon$ $\therefore \sigma = E\varepsilon$ —— (i)

For shear stress & strain; $\tau \propto \phi$ $\therefore \tau = G\phi$ —— (ii)

For volumetric stress & strain; $\sigma_v \propto \varepsilon_v$ $\therefore \sigma_v = K\varepsilon_v$ —— (iii)

Now, combining eqn (i), (ii) & (iii)

$E = 2G(1+\mu)$ —— ①	Where;
$E = 3K(1-2\mu)$ —— ②	E = young modulus of elasticity
$E = \frac{9KG}{3K+G}$ —— ③	G = Shear modulus of rigidity
	K = bulk modulus
	μ = Poisson's ratio.

Notes: → Young. modulus inversely proportional to temperature.

→ **Modulus of elasticity (E)** $= \dfrac{\text{Stress}}{\text{Strain}}$

→ $E > G$

→ **Modulus of Rigidity (G)** $= \dfrac{\text{shear stress}}{\text{shear strain}}$

→ **Bulk modulus (k)** $= \dfrac{\text{Direct stress}}{\text{Volumetic strain}}$

→ $\mu_{rich} > \mu_{lean}$

Structural property.

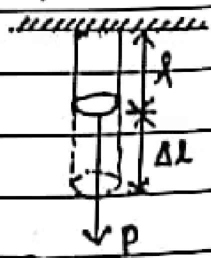
① **stability** Ability to remain in equilibrium.
② **Strength** Ability to resists the load without failure.
③ **stiffness** Ability to resists deformation of body.

Strain Energy.

→ Strain energy is **Stored energy** within elastic limit which resist the deformation.

→ **strain energy** due to A.F.	$= \int \dfrac{P^2}{2AE} \, dx$	where; P = Axial force, A = cross-sectional are
→ " " " " S.F	$= \int \dfrac{V^2}{2AG} \, dx$	V = shear force
→ " " " " B.M	$= \int \dfrac{M^2}{2EI} \, dx$	M = Bending moment, I = moment of inertia
→ " " " " T.F	$= \int \dfrac{T^2}{2GJ} \, dx$	T = Torsional force, J = polar MOI

Deformation of member



① **Due to axial load** $\boxed{\Delta l = \dfrac{PL}{AE}}$

② **Teparing bar** $\boxed{\Delta l = \dfrac{4Pl}{\pi d_1 d_2 E}}$

③ **Trapezoidal section** $\boxed{\Delta l = \dfrac{Pl}{Et(b_2 - b_1)} \times \ln\left(\dfrac{b_2}{b_1}\right)}$

* **self wt** $\boxed{\Delta l = \dfrac{WL}{}}$

Center of Gravity

→ C.G of a body is that point through which the resultants of the parallel forces formed by the weight of all particles of the body.

→ It is center of mass.

→ It may also lies outsides of the body.

→ $C.G = (x, y) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}\right), \left(\frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$

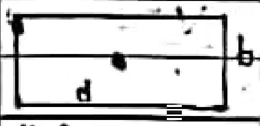
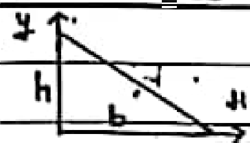
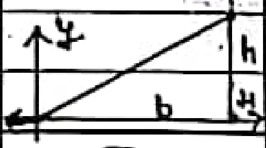
Centroid

→ The plane figure having only area but not mass.

→ The geometrical center of area such figure is called centroid. → It is the center point of area.

→ $Centroid = (\bar{x}, \bar{y}) = \left(\frac{A_1x_1 + A_2x_2}{A_1 + A_2}\right), \left(\frac{A_1y_1 + A_2y_2}{A_1 + A_2}\right)$

Centroid of different geometrical figure

Shape	Figure	Area	$\bar{x}$	$\bar{y}$
① Rectangle		$b \times d$	$\frac{b}{2}$	$\frac{d}{2}$
② Triangle		$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
		$\frac{1}{2}bh$	$\frac{2b}{3}$	$\frac{h}{3}$
③ Circle		πr^2	$\frac{d}{2}$	$\frac{d}{2}$
④ Semi-Circle		$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$

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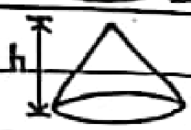
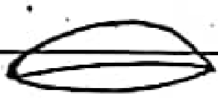
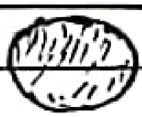
(5) Quarter circle	Figure	Area	$\bar{x}$	$\bar{y}$
		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
(6) solid cone or pyramid			$\frac{d}{2}$	$\frac{h}{4}$
(7) Hollow cone or pyramid			$\frac{d}{2}$	$\frac{h}{3}$
(8) solid hemisphere			$\frac{d}{2}$	$\frac{3r}{8}$
(9) Hollow hemisphere			$\frac{d}{2}$	$\frac{r}{2}$
(10) sphere			$\frac{d}{2}$	$\frac{d}{2}$

Moment of Inertia

→ The second moment of area is called moment of inertia.

→ The second moment of mass is called mass moment of inertia.

→ The moment of inertia is the physical property of a rigid body that determines the amount of torque needed to rotated the body in given axis.

→ It is defined as the sum of the product of the area of each particle in the body & square of its distance from the axis of rotation.

→ Mathematically; x-axis, $I_{xx} = \int y^2 dA$

Y-axis, $I_{YY} = \int x^2 dA$

→ It resist deformation. so the greater value of 'I' is better structural stability.

→ Its value is always positive & its unit is m^4.

→ Unit for mass moment of inertia is $kg\text{-}m^2$.

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Moment of inertia of different shape

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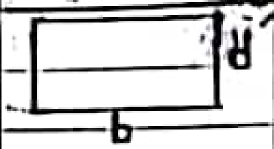
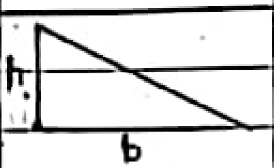
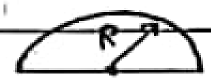
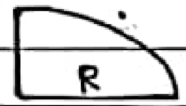
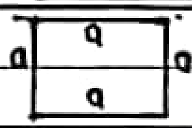
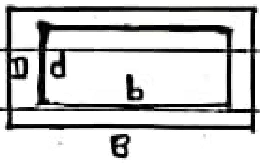
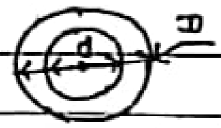
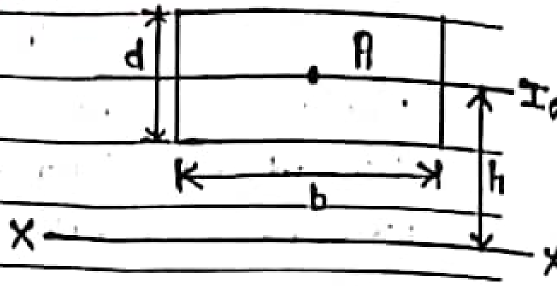
Shape	Figure	About C.G. $(I_G)_X$	About C.G. $(I_G)_Y$	About base I_{XX}	About base I_{YY}
① Rectangle		$\frac{bd^3}{12}$	$\frac{db^3}{12}$	$\frac{bd^3}{3}$	$\frac{db^3}{3}$
② Triangle		$\frac{bh^3}{36}$	$\frac{hb^3}{36}$	$\frac{bh^3}{12}$	$\frac{hb^3}{12}$
③ Circle		$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$		
④ Semi-circle		$0.11R^4$	$\frac{\pi R^4}{8}$		
⑤ Quarter circle		$0.055R^4$	$0.055R^4$		
⑥ Square		$\frac{a^4}{12}$	$\frac{a^4}{12}$	$\frac{a^4}{3}$	$\frac{a^4}{3}$
⑦ Hollow rectangle		$\frac{BD^3-bd^3}{12}$	$\frac{DB^3-db^3}{12}$		
⑧ Hollow circle		$\frac{\pi(D^4-d^4)}{64}$	$\frac{\pi(D^4-d^4)}{64}$		
⑨ Trapezoidal		$\frac{h^3(a^2+4ab+b^2)}{36(a+b)}$			
⑩ Ellipse		$\frac{\pi}{64}ab^3$	$\frac{\pi}{64}ba^3$		

Axis of symmetry → such line or plane that divides the whole body into two equal parts. → It passes always centroid.

Parallel Axis Theorem

$I_{XX} = (I_G)_X + Ah^2$

$= \frac{bd^3}{12} + Ah^2$



$\boxed{I_{YY} = (I_G)_Y + Ah^2}$

$= \frac{db^3}{12} + Ah^2$

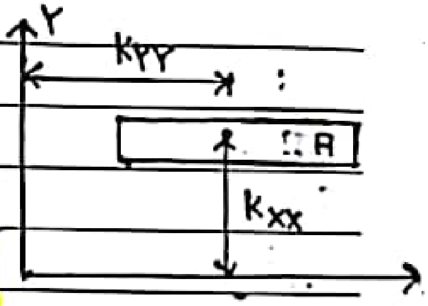
Radius of Gyration

→ Radius of gyration of any lamina (2D body) about a given axis may be defined as the distance from a given axis at which all the element parts of the lamina would have to be placed so as not to alter MOI about given axis.

Mathematically;

$I_{XX} = A \times K_{XX}^2$ ∴ $\boxed{K_{XX} = \sqrt{\frac{I_{XX}}{A}}}$

Similarly; Y-axis ∴ $\boxed{K_{YY} = \sqrt{\frac{I_{YY}}{A}}}$



Polar Moment of Inertia

→ Moment of inertia perpendicular to the axis of plane figure is called polar moment of inertia. → $\boxed{I_{ZZ} = I_{XX} + I_{YY}}$

Flexure Equations or Bending Equations

Assumptions

① Plane of cross-section of beam remains plane before & after bending

② Materials of beam obeys Hooke's law.

③ Materials of beam is homogenous.

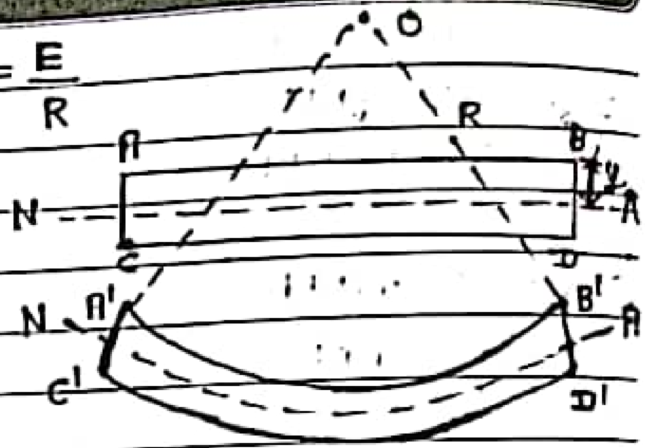
④ Beam bends to circular arc.

⑤ Load acts perpendicular to the beam axis.

⑥ Radius of curvature is large, i.e. deflection is small.

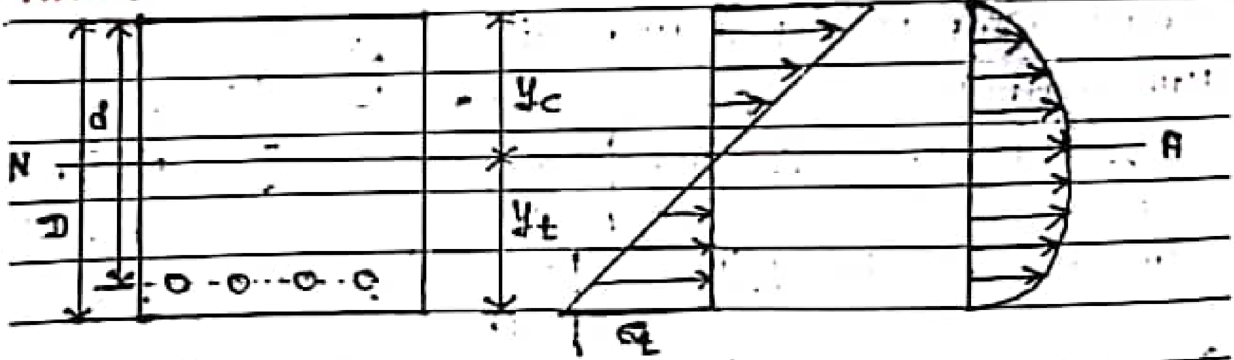
⑦ Direction of [illegible] is same.

Mathematically; $\frac{M}{I_1} = \frac{\sigma}{y} = \frac{E}{R}$



M = Bending moment
= Moment of Resistance
I = Moment of Inertia
σ = Bending stress
R = Radius of curvature
E = young modulus of elasticity
y = Distance from neutral axis to extreme fiber.

Note:



d = effective depth — Bending stress diagram — Shear stress diagram

→ Bending stress in neutral axis is zero.
→ Shear stress in neutral axis is maximum.
→ At Neutral axis both bending stress & strain are zero.

Neutral Axis An imaginary horizontal line that passes through C.g of transverse cross-section & at which bending stress is zero & shear stress is maximum.

Section Modulus

Section modulus of a beam is the quantity obtained by dividing the moment of inertia of the beam, about its C.g.

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by the distance of extreme fiber from the neutral axis.

→ Mathematically; $\boxed{Z = \frac{I}{y}}$

from bending eqn,

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \frac{M}{\sigma} = \frac{I}{y} \Rightarrow M = \frac{I}{y} \times \sigma \Rightarrow \boxed{M = Z \times \sigma}$$

Section Modulus for standard section

① Rectangular section, $Z = \frac{BD^2}{6}$

② Hollow rectangular section, $Z = \frac{1}{6}\left(\frac{BD^3 - bd^3}{D}\right)$

③ Circular section, $Z = \frac{\pi D^3}{32}$

④ Hollow circular section, $Z = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$

Pure Bending

→ Bending of beam due to the constant maximum bending moment when shear force is zero is called pure bending.

→ In figure, pure bending in CD portion.

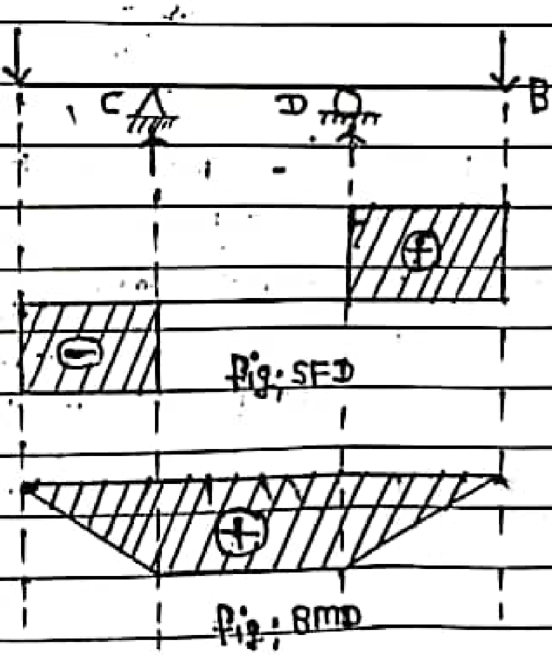


Fig: SFD

Fig: BMD

Note : → from SFD, verticle stirrup..... are designed.

→ From BMD, longitudinal bars are designed.

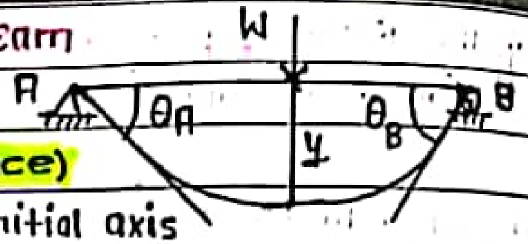
→ The spacing of shear reinforcement is more at center as compared to end (support) of beam.

→ Bending stress distribution is triangular.

→ shear stress distribution is parabolic.

→ Most efficient section for flexure member is I-section.

Deflection of Beam



Deflection

The vertical ordinate (distance) between deflection curve & initial axis is called deflection.

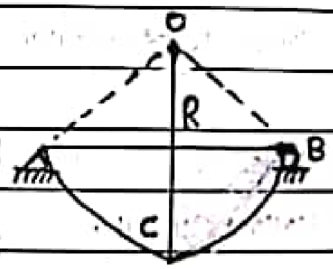
→ In figure, 'y' is the maximum deflection.

Slope

Slope of a beam at a point is an angle made by the tangent to reference axis at that point.

→ θ_A & θ_B are the slope at A & B.

Curvature



Here, the distance 'OC' is radius of curvature (R) & the inverse of it's is called curvature.

→ Mathematically: $\boxed{\text{Curvature} = \frac{1}{R}}$

Relationship between slope, deflection & Radius of Curvature

$$\boxed{M = EI\frac{d^2y}{dx^2}}$$

→ This is the differential eqn of elastic curve.

→ Equations of elastic curve.

Where; M = Bending moment

E = Young modulus of elasticity.

I = Moment of inertia.

$\frac{d^2y}{dx^2}$ = double derivative = Area under the curve.

Method of determining slope & deflection of beam

① Double integration method

② Moment area method

③ Conjugate beam method.

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Note: → Deflection is maximum at center of S.S.B

→ slope is zero at center of S.S.B.

→ slope is maximum at support of S.S.B.

→ The rate of change of strain energy is called deflection.

→ The rate of change of deflection is called slope.

→ The rate of change of shear force is called intensity of loading

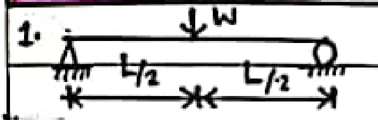
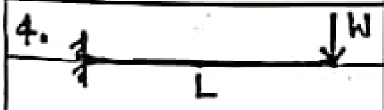
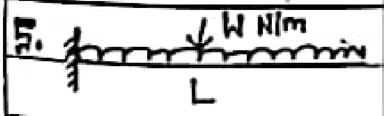
→ The rate of change of bending moment is called shear force

→ EI = flexure Rigidity → EA = Axial Rigidity

→ GA = Shear Rigidity → GJ = Torsional Rigidity

→ The rate of change of slope (curvature) is called Bending moment

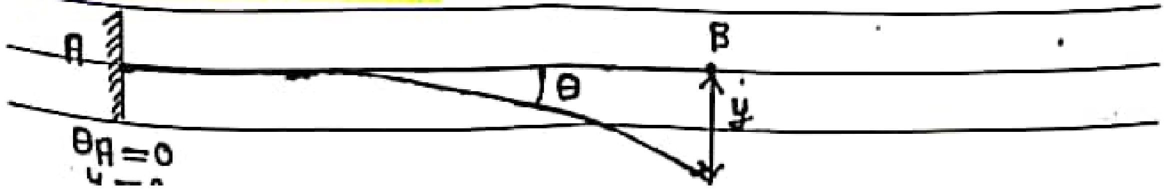
Slope & deflection of beam bending

Beam & Loading	Slope	Max. deflection	Max. moment
1. 	$\frac{WL^2}{16EI}$ at support	$\frac{WL^3}{48EI}$ at center	$\frac{WL}{4}$
2. 	$\frac{WL^3}{24EI}$ at support	$\frac{5WL^4}{384EI}$ at center	$\frac{WL^2}{8}$
3. 	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
4. 	$\frac{WL^2}{2EI}$ at free end	$\frac{WL^3}{3EI}$ at free end	WL
5. 	$\frac{WL^3}{6EI}$ at free end	$\frac{WL^4}{8EI}$ at free end	$\frac{WL^2}{2}$

Note: → In case of Cantilever beam slope at support always zero

→ In case of Cantilever beam deflection at support is always zero.

→ In case of Simply supported Beam with symmetric loading, slope at center always zero.



Point of Contraflexure / inflection :-

→ The point at which bending moment is zero & through which bending moment diagram changes its sign from positive to negative or vice-versa.

→ It is also known as Virtual hinge.

Nature of SFD & BMD of simply supported Beam

Load	SFD	BMD
P	constant	Linear
w	Linear	Parabolic
w	Parabolic	Cubic

Nature of SFD & BMD of Cantilever beam

Load	SFD	BMD
W	Constant	Linear
W	Linear	parabolic

Note : At the point of maximum bending moment, shear force changes the sign (zero except poind load)

→ At point where shear force is zero, BM will be either maximum or minimum.

→ In case of point loading, Bm is maximum at the point of application of load.

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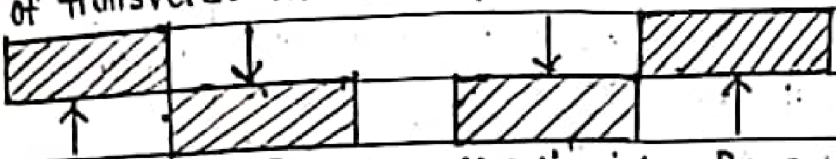
Mechanics of Beam

① Shear Force

→ Shear force is the force acting tangential to the cross-secti of member. → It acts perpendicular to the axis of member.

→ Shear force of a section may be defined as the algebraic sum of transverse force acting to either side of the section.

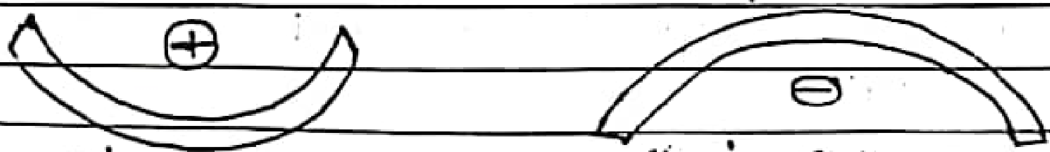


Positive shear force Negative shear force

→ The diagram of showing the variation of shear force is called SFD.

② Bending Moment

→ Bending moment of a section may be defined as the algebrai sum of moments of external loads acting to their either sides of section. → The diagram showing the variation of bending moment along the span is called bending moment diagram.



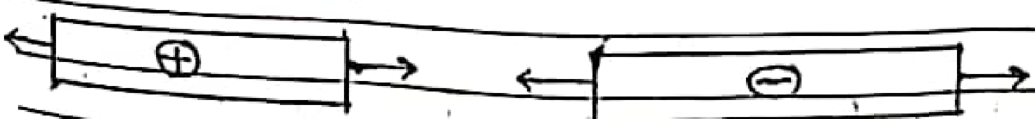
Sagging curve Hogging curve

Positive Bending moment Negative Bending Moment

③ Axial Force

→ It is a force acting normal to the cross-section of member.

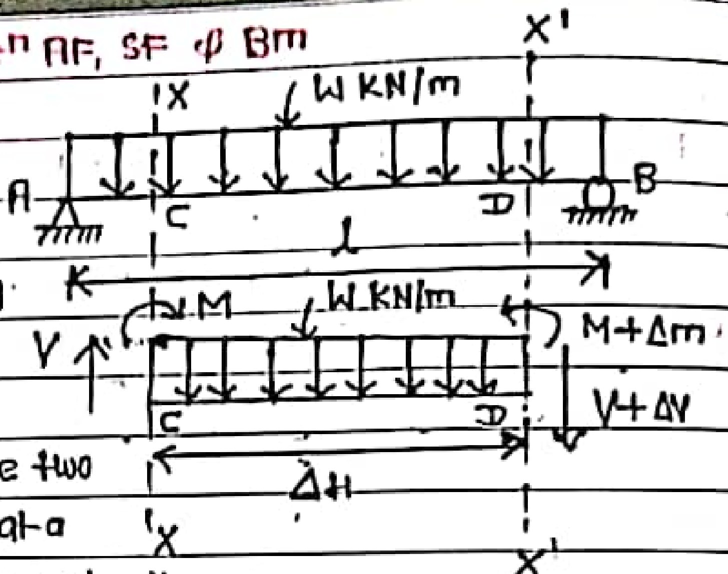
→ Axial force is the algebraic sum of all the forces acting along the longitudinal axis of the member on either sides of considered section.



Positive Axial Force Negative Axial Force.

Relationship betn AF, SF & Bm

Let us consider a simply supported beam AB carrying uniformly distributed load (UDL) w per unit length.



let C & D be the two points on the beam at a distance ΔH from each other.

The shear force & bending moment at 'C' is denoted by V & M respectively.

Therefore, the SF & Bm at point D will be V+ΔV & M+Δm respectively as shown in figure.

Relation betn load & shear force

Algebraic sum of verticle force is equal to zero.

$\Sigma F_y = 0 \; (\uparrow +)$

or, $V - w \cdot \Delta H - (V + \Delta V) = 0$

or, $\cancel{V} - w \cdot \Delta H - \cancel{V} - \Delta V = 0$

or, $-\Delta V = w \cdot \Delta H$

$\therefore \; w = -\dfrac{\Delta V}{\Delta H}$

limit $\Delta H \longrightarrow 0$

$\dfrac{dV}{dH} = -w$ —— (i)

∴ The rate of change of shear force is equal to area under the load curve.

on integration; $\int_A^B dV = -\int_A^B w \, dH$

$V_B - V_A = -$(Area under the load curve)

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i.e. $\frac{dV}{dx} = -W$ (Area under the load curve)

Relation betⁿ S.F & BM

Taking moment at point D

$\Sigma M_D = 0$ (↻+)

or, $M + V.\Delta x - W.\Delta x.\frac{\Delta x}{2} - (m + \Delta m) = 0$

or, $\cancel{m} + V.\Delta x - \frac{W\Delta x^2}{2} - (\cancel{m} + \Delta m) = 0$

Neglecting square term

or $V.\Delta x - \Delta m = 0$

or $\Delta m = V.\Delta x$

$\therefore \frac{\Delta m}{\Delta x} = V$

Taking limit

$\lim_{\Delta x \to 0} \frac{\Delta M}{dx} = V$

$\frac{dm}{dx} = V$ —— (11)

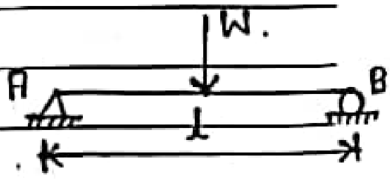
$\therefore$ **The rate of change of bending moment is equal to S.F.**

Load

It may be defined as an agent which produce or tends to produce destroys or tends to destroys motion. **Unit: N or kN**

Types of Load

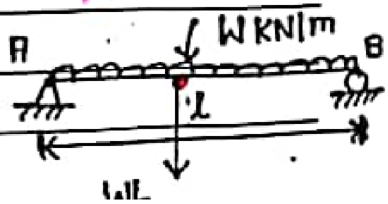
① Point Load: → It is a load which is **assumed to act at a point.**



→ It is also called **concentrated load.**

→ It unit is N or kN.

② Uniformly Distributed Load (UDL)



→ It is a load which is **distributed uniformly over the length of beam.**

→ The unit of udl is N/m or kN/m.

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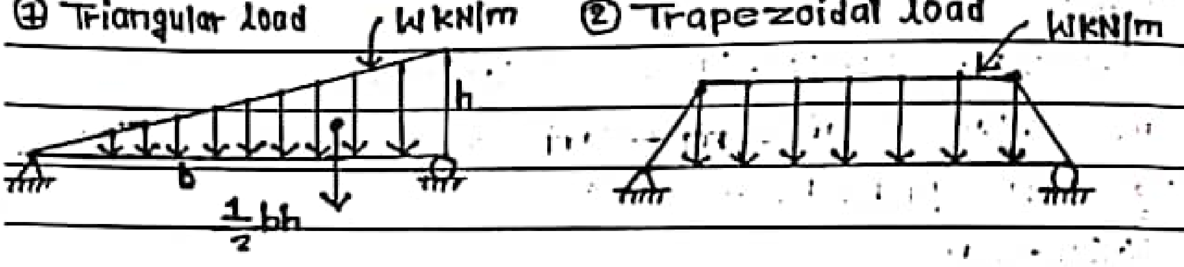
③ Uniformly Varying Load (UVL) → It is a load which is distributed non-uniformly over the length of beam.

→ The unit of UVL is N/m or KN/m.

→ It is two types.

① Triangular load ② Trapezoidal load



Types of support

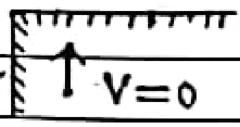
① Simple support

→ A simple support offers only a Vertical reaction.

→ Horizontal moment or rotation of the member are not prevented at this support. → No of reaction = 1

→ Example : masonry Wall → DOF = 2

→ Rotation (θ) = ✓ → Hz. displacement = ✓



→ Vertical displacement = (0) ✗

② Roller support

→ It is similar to simple support

→ The support which provided with roller.

→ Example : Roller bearing of bridges, trusses.

→ Rotation = ✓ → Hz. displacement = ✓ → V. displacement = 0

→ No. of reaction = 1 → DOF = 2

③ Hinged / Pin support

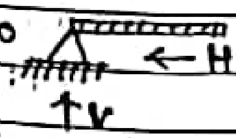
→ The support which provided with frictionless pin.

→ Prevented both horizontal & Vertical movement.

→ Example : single riveted lap joint.

→ Rotation = ✓ → Hz. displacement = 0

→ V. displacement = 0



→ No. of reaction = 2 → DOF = 1

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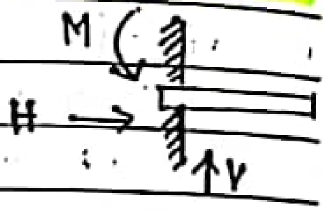
④ Fixed support

The support which provided resistance to rotation about the support & to the movement in verticle & horizontal direction is called fixed support.



→ No. of reaction = 3 → DOF = 0

→ Rotation = 0 → Hz. displacement = 0

→ V. displacement = 0

→ Example : Beam & Column of framed structure.

Beam

It is the horizontal member of structure. → Beams are normally placed in horizontal position & loaded with vertical loads.

Types of Beam

① Simply supported Beam : Used : Load bearing wall.

→ It is a beam with one end hinged & another end roller supports.

② Cantilever Beam

→ It is a beam with one end fixed & another end free.

③ Propped Cantilever Beam

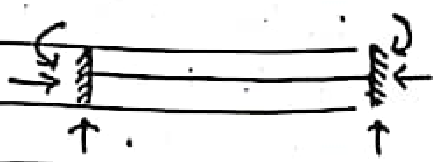
→ It is a beam with one end fixed & another end roller supports.

→ It is generally used in bridge construction.

④ Fixed Beam

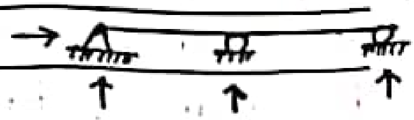
→ It is a beam with both end fixed.

→ It is generally used in RCC structure.

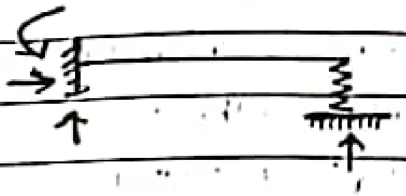


⑤ Continuous Beam

→ It is a beam with more then two support.

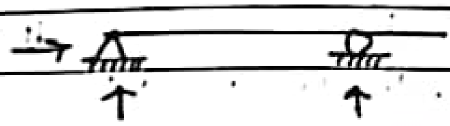


⑥ Elastic propped Beam



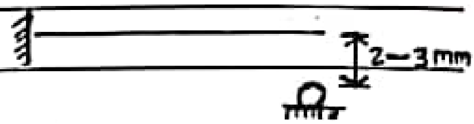
→ It is a beam with one end fixed & another end elestic materials is used.

⑦ Over-hanging Beam



→ It is a beam at least one end out of support.

⑧ Sink propped Beam



→ It is a beam with one end fixed & another end roller support with some Varticle distance.

Static Equilibrium A particle or rigid is said to be in static equilibrium if it is at rest.

→ Static equilibrium means the net force & net moment acting on a body is zero.

Condition of static Equilibrium

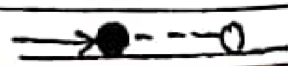
$\Sigma f_x = 0 \quad \Sigma f_y = 0 \quad \Sigma M = 0$ (2D)

(3D) $\Sigma f_x = 0 \quad \Sigma f_y = 0 \quad \Sigma f_z = 0 \quad \Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$

Types of Equilibrium

① Stable if the body returns back to it's original position when it is slidely displaced from it's position of rest, then it is called Stable equilibrium.

② Neutral A body said to be in neutral equilibrium if it is occupies a new position & remains at rest in the position when it is slidely displaced from it's original position.



③ Unstable: opposite to the stable equilibrium.

Static Determinate & Indeterminate structure

→ if the unknown internal forces & reaction can be solved only using static equilibrium equation, then the structure said to be determinate, otherwise indeterminate.

Frame	Truss	Determincy
$3m+r = 3j+c$	$m+r = 2j$	Statically determinate
$3m+r > 3j+c$	$m+r > 2j$	Statically indeterminate
$3m+r < 3j+c$	$m+r < 2j$	Unstable

Types of Structure	Where;
① $m = 2j-3$ → Determinate structure	m = no. of members
② $m > 2j-3$ → Indeterminant structure	j = no of joints
③ $m < 2j-3$ → Unstable structure	r = no of reaction
	c = no of loop

Degree of Static Indetermincy

① External static indetermincy $(D_E) = r-3$

② Internal static indetermincy $(D_I) = 0$ for beam

$= 3c$ for frame

$= m-(2j-3)$ for truss

Note:

① $m-(2j-3) = 0$ → internally determinant

② $m-(2j-3) > 0$ → " indeterminant

③ $m-(2j-3) < 0$ → " determinant but unstable.

Total degree of static indetermincy $\boxed{(D_S) = D_E + D_I}$

Simple strut Theory

A Structure is subjected to an axial compressive force is called strut. if the strut is verticle (90° to the horizontal) is known as column.

→ strut may have it's one end or both end fixed rigidly or hinged or pin jointed, while column will have both ends fixed.

Compression member

Column → Column is a compression member which carries axial compression in from structure.

→ Most economical section of column is tubular section.

Classification of Column

① $\frac{l_{eff}}{d_{min}} \leq 3$ → Pedestal

② $3 < \frac{l_{eff}}{d_{min}} \leq 12$ → Short column, $10 < \frac{l_{eff}}{r_{min}} < 40$

③ $\frac{l_{eff}}{d_{min}} > 12$ → Long column, $\frac{l_{eff}}{r_{min}} > 40$

where, l_{eff} = effective length of column.

d_{min} = least lateral dimension of column.

r_{min} = least radius of gyration.

For steel column

① $\frac{l_{eff}}{r_{min}} < 50$ → short column

② $50 < \frac{l_{eff}}{r_{min}} < 250$ → Intermediate column

③ $\frac{l_{eff}}{r_{min}} > 250$ → Long column

Modes of failure of Column

Column type	Short	Long	Intermediate
Mode of failure	Crushing	Buckling	Crushing & buckling combined

Strut → A long structural member subjected to compression load.
→ member of truss which carries only axial compression is known as strut. → Tension member of truss is known as tie.

Slenderness Ratio

It is the ratio of effective length of column to the least radius of gyration. Mathematically,

$$\text{Slenderness Ratio } (\lambda) = \frac{l_{eff}}{r}$$

Where,
L = total length
l_{eff} = effective length
r = least radius of gyration $(K) = \sqrt{\frac{I}{A}}$
I = Moment of inertia
A = X-sectional area.

Euler's Theory for Crippling/Buckling Load

→ Load under which column fails due to the buckling is known as crippling load or critical load. → This theory valid for only long column
→ Crippling or critical load

$$(P) = \frac{C\pi^2 E I_{min}}{(l_{eff})^2}$$

Where,
P = Crippling load
E = Young modulus of elasticity for the materials.
C = Factor announcing for the end condition.
I_{min} = The least moment of area of section (mm^4 or m^4)
l_{eff} = effective length of column.
→ It depends on end conditions.

Assumption → The materials of column is homogenous. → Initially straight → obey's hookes law. → uniform lateral dimension. → self wt. of column neglected. → end of column are frictionless → column fails by buckling only.

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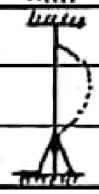
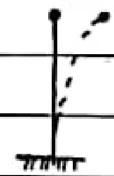
End Condition

S.N	End Condition	Figure	Effective length	Crippling load
1	Both end hinged		$l_{eff} = L$	$\frac{\pi^2 EI}{L^2}$
2.	Both end Fixed		$l_{eff} = \frac{L}{2}$	$\frac{4\pi^2 EI}{L^2}$
3.	One end fixed & other hinged		$l_{eff} = \frac{L}{\sqrt{2}}$	$\frac{2\pi^2 EI}{L^2}$
4.	One end fixed & other free		$l_{eff} = 2L$	$\frac{\pi^2 EI}{4L^2}$

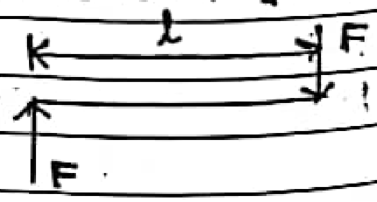
Moment The moment of force about a given point is the product of magnitude of the force & the perpendicular distance of the point from the line of action of the force.

Mathematically; $\boxed{M = F \times d}$

Torque It is to be consider that that the point & the line of action of force should be in the same plane otherwise torque is produced.

Couple The combination of two equal & opposite force acting along different lines of action on a rigid body is called Couple.

→ $\boxed{\text{Couple} = F \times l}$



Deformation shift the particles with in the elastic limit.
Displacement shift beyond the elastic limit. i.e the large deflect which can be seen by hecked eye.

Materials

① Brittle materials It is the materials fails suddenly without warning when stressed beyond their strength.
→ The compression test is commonly used for testing the brittle materials.

② Homogenous Materials A materials having similar property throughout it's volume is called homogeneous materials.

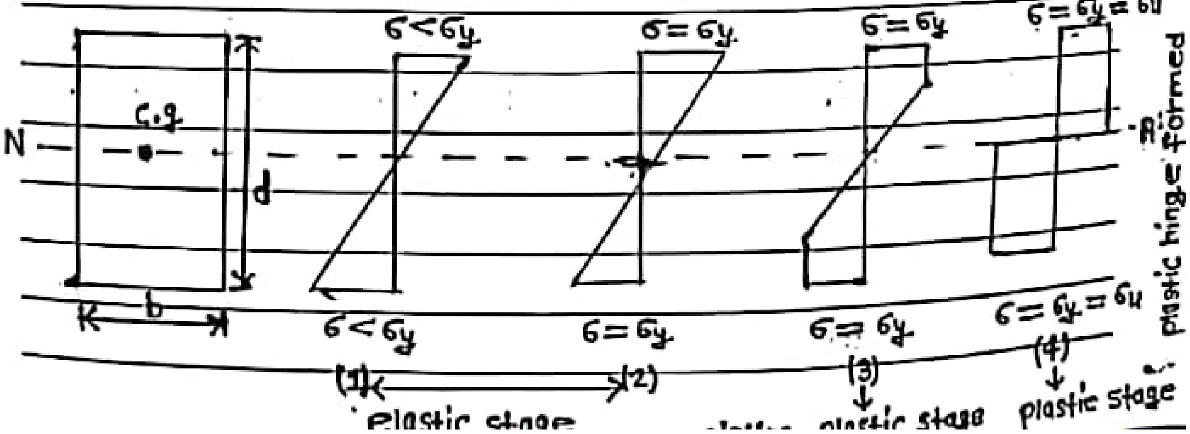
③ Isotropic materials A materials having identical property in all the direction is called isotropic.

④ Orthotropic materials: The materials which have different properties in different direction is called orthotropic materials.

⑤ Toughness materials It absorb energy at high stress without failure. → Impact test is done to estimate the toughness of materials.

⑥ Visco-elastic materials → Time dependent stress-strain relation. → The material which shows different stress-strain relation at different time.

Internal Effects of Loading



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where: σ_y = yield stress

σ = working stress

σ_u = ultimate strength

Stage – I: The externally applied load is such that stress developed is less then the yield stress.

Stage – II: if load goes increasing, at the some value of load, the stress in extreme fiber reach yield stress.

Stage – III: With further increasing load, the yield stress at extreme fiber remains constant but the yielding spreads into the inner fibers.

Stage – IV: Again, if applied load goes on increasing, at some value of load all the fibers will be subjected to yield stress.

→ The load carried at this stage is called ultimate load, & the strength of materials corresponding to the ultimate load is called ultimate strength.

Structural members

(1) Beam → members subjected to bending or flexure.

(2) Ties/Link → members subjected to only axial tension.

(3) Struts → members subjected to only axial compression.

(4) Truss → Group of ties & struts. → It consists of pin joint that transfer only the axial force. → It means only axial force developed in truss as an internal forces.

(5) Rigid Frame → It consists of rigid joints. i.e, moment resisting joint, that transfer all the internal forces such as axial, shear & bending moment.

Characteristics of SFD & BMD

Diagram \ Load	Point load	UDL	UVL
S.F.D	Hz. or Verticle line	Triangle	Sq. parabola
B.M.D	Triangle	Sq. Parabola	Cu. parabola

Simply supported Beam

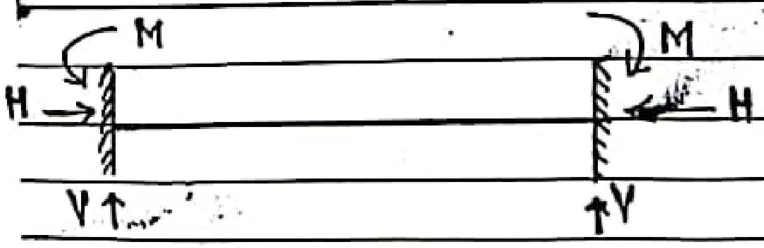
Loading	SF	BM
① Point load at mid span	Max at support $= W/2$	$M_{max} = \frac{Wl}{4}$ at center
② UDL	$V_x = Wl/2 - Wx$ max at support $Wl/2$	$M_x = Wx \times x/2 - \frac{Wl}{2}x$ $M_{max} = \frac{Wl^2}{8}$ at center
③ UVL	$V_x = \frac{Wl}{4} - \frac{Wx^2}{l}$ $V_{max} = \frac{Wl}{4}$ at support	$M_{max} = \frac{Wl^2}{12}$

Cantilever Beam

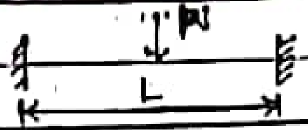
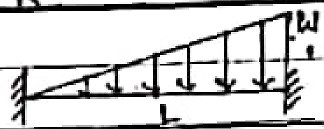
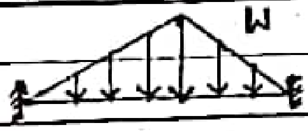
Loading	S.F.	BM
moment at end of beam	zero	M
point load at free end	$V_{max} = Wl$ (constant)	$M_{max} = Wl$ at support
UDL	$V_{max} = Wl$ at support	$M_{max} = \frac{Wl^2}{2}$ at support

Fixed Beam

Loading	SFD	BMD
① point load at center	$V_{max} = W/2$	$M_{max} = Wl/8$
② UDL	$V_{max} = \frac{Wl}{2}$	$M_{max} = \frac{Wl^2}{12}$



fig; Fixed beam

Nature of load	BM(max)	Max. Deflection
① 	$\frac{WL}{8}$	$\frac{WL^3}{192EI}$
② 	$\frac{WL^2}{12}$, $\frac{WL^2}{24}$ (center)	$\frac{WL^4}{384EI}$
③ 	$\frac{WL^2}{30}$, $\frac{WL^2}{20}$	$\frac{5WL^4}{96EI}$, $\frac{WL^4}{32EI}$ (center)
④ 	$0.0013WL^2$	$\frac{0.73WL^4}{384EI}$